# A Method of Deduction of Pressure Distribution on an Internal Plane Surface from the Observed Topographical Change(Part ?).



| | |
|---|---|
| 著者 | Nakamura Saemon Taro, Sima Hiromu |
| 雑誌名 | Science reports of the Tohoku University. Ser. 5, Geophysics |
| 巻 | 3 |
| 号 | 2 |
| ページ | 99-101 |
| 発行年 | 1951-09 |
| URL | http://hdl.handle.net/10097/44466 |

# A METHOD OF DEDUCTION OF PRESSURE DISTRIBUTION ON AN INTERNAL PLANE SURFACE FROM THE OBSERVED TOPOGRAPHICAL CHANGE (PART I)

Saemon Taro NAKAMURA* and Hiromu SIMA

*Institute of Geophysics, Faculty of Science, Tôhoku University*

(Received February 19, 1951)

K. SEZAWA calculated the topographical change due to an internal pressure. He assumed a cavity, whose radius is $a$ and on the internal surface of the cavity pressure $p$ is acting and the outer medium near the cavity is in a plastic state from radius $a$ to $b$.

He get the final result that at the earth's surface ($z=0$) the horizontal and vertical displacements are given by

$$u = \frac{2B(\lambda+2\mu)}{\lambda+\mu} \frac{r}{(H^2+r^2)^{3/2}}, \quad \dots\dots\dots\dots\dots\dots\dots\dots \quad (1)$$

and

$$v = \frac{2B(\lambda+2\mu)}{\lambda+\mu} \frac{H}{(H^2+r^2)^{3/2}},$$

where $r$ is the horizontal distance of the point considered on the earth's surface from the cavity, $H$ the depth of the cavity, $\lambda$ and $\mu$ LAMÉ's constants and $B$ is given by

$$B = \frac{pb^3}{4\mu} + \frac{kb^3}{2\mu} \log \frac{b}{a}. \quad \dots\dots\dots\dots\dots\dots\dots\dots \quad (2)$$

In equation (2) $k$ is the maximum shearing stress.

Ignoring details of $B$, put

$$A = \frac{2B(\lambda+2\mu)}{\lambda+\mu}, \quad \dots\dots\dots\dots\dots\dots\dots\dots \quad (3)$$

and represent the strength of the pressure by $A$.

If the distribution of such a pressure on an internal horizontal surface is given, the resultant displacement on the earth's surface can be given by

$$u = \iint \frac{Ar}{(H^2+r^2)^{3/2}} \frac{x-x'}{r} dx'dy',$$

$$v = \iint \frac{Ar}{(H^2+r^2)^{3/2}} \frac{y-y'}{r} dx'dy', \quad \dots\dots\dots\dots\dots\dots\dots\dots \quad (4)$$

and

$$w = \iint \frac{AH}{(H^2+r^2)^{3/2}} dx'dy'.$$

C. TSUBOI proposed an ingeneous method to estimate the distribution of anomaly of

* Now Institute of Physics, Faculty of Science, Kumamoto University.

density assumed to be distributed on a surface at a certain depth when the anomaly of gravity on the earth's surface is given. It is based on that the distribution of the anomalies of density and gravity are expanded in FOURIER's double series and that the corresponding terms in the series are compared each others.

Applying the same procedure to SEZAWA's result, we can easily estimate the distribution of pressure represented by $A$ in equation (3).

Put

$$\begin{aligned} A = A_0 &+ A_{om} \cos my' + A_{no} \cos nx' + B_{om} \sin my' + B_{no} \sin nx' \\ &+ \Sigma A_{nm} \cos nx' \cos my' + \Sigma A'_{nm} \cos nx' \sin my' \\ &+ \Sigma B_{nm} \sin nx' \cos my' + \Sigma B'_{nm} \sin nx' \sin my', \quad \ldots\ldots\ldots\ldots (5) \end{aligned}$$

then the corresponding displacement at the earth's surface is given by

$$\left.\begin{aligned} u = {} & \Sigma A_{no} I_1^{(n)} \sin nx' - \Sigma B_{no} I_1^{(n)} \cos nx' + \Sigma A_{nm} I_3^{(n)} \sin nx' \cos my' \\ & + \Sigma A'_{nm} I_3^{(n)} \sin nx' \sin my' - \Sigma B_{nm} I_3^{(n)} \cos nx' \sin my' \\ & - \Sigma B'_{nm} I_3^{(n)} \cos nx' \sin my', \\ v = {} & \Sigma A_{om} I_2^{(m)} \sin my' - \Sigma B_{om} I_2^{(m)} \cos my' + \Sigma A_{nm} I_3^{(m)} \cos nx' \sin my' \\ & - \Sigma A'_{nm} I_3^{(m)} \cos nx' \cos my' + \Sigma B_{nm} I_3^{(m)} \sin nx' \sin ny' \\ & - \Sigma B'_{nm} I_3^{(m)} \sin nx' \cos my', \\ w/H = {} & A_0 I_0 + \Sigma (A_{no} \cos nx' + B_{no} \sin nx') I_1 \\ & + \Sigma (A_{om} \cos my' + B_{om} \sin my') I_2 \\ & + \Sigma (A_{nm} \cos nx' \operatorname{con} my' + A'_{nm} \cos nx' \sin my' \\ & + B_{nm} \sin nx' \cos my' + B'_{nm} \sin nx' \sin my') I_3, \end{aligned}\right\} \ldots\ldots (6)$$

where $I$'s represent the following integrals,

$$\left.\begin{aligned} & I_0 = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \frac{d\xi\, d\eta}{(H^2+r^2)^{3/2}} = \frac{2\pi}{H}, \quad I_1 = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \frac{\cos n\xi\, d\xi\, d\eta}{(H^2+r^2)^{3/2}} = 2\pi e^{-nH}/H, \\ & I_2 = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \frac{\cos my\, d\xi\, d\eta}{(H^2+r^2)^{3/2}} = 2\pi e^{-nH}/H, \\ & \qquad\qquad I_1^{(n)} = -\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \frac{\xi \sin n\xi\, d\xi\, d\eta}{(H^2+r^2)^{3/2}} = -2\pi e^{-nH}, \\ & I_2^{(m)} = -\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \frac{\eta \sin m\eta\, d\xi\, d\eta}{(H^2+r^2)^{3/2}} = -2\pi e^{-mH}, \\ & I_3 = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \frac{\cos n\xi \cos m\eta\, d\xi\, d\eta}{(H^2+r^2)^{3/2}} = \frac{2\pi m}{H} e^{-nH} \\ & \qquad\qquad - \pi \int_{mH}^{\infty} \frac{J_1(\sqrt{y'^2 - m^2 H^2})}{\sqrt{y'^2 - m^2 H^2}} e^{-\frac{n}{m} y'} dy', \end{aligned}\right\} \ldots\ldots (7)$$

$$\left.\begin{aligned}I_3^{(n)} &= -\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \frac{\xi \sin n\xi \cos m\eta\, d\xi\, d\eta}{(H^2+r^2)^{3/2}} = -2\pi m e^{-nH} \\ &\qquad - \frac{\pi}{m}\int_{mH}^{\infty} \frac{J_1(\sqrt{y'^2-m^2H^2})}{\sqrt{y'^2-m^2H^2}}\, y e^{-\frac{n}{m}y'} dy', \\ I_3^{(m)} &= -\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \frac{\cos n\xi \cdot \eta \cdot \sin m\eta\, d\xi\, d\eta}{(H^2+r^2)^{3/2}} = -2\pi n e^{-mH} \\ &\qquad + \frac{\pi}{n}\int_{nH}^{\infty} \frac{J_1(\sqrt{x'^2-n^2H^2})}{\sqrt{x'^2-n^2H^2}}\, x e^{-\frac{m}{n}x'} dx'.\end{aligned}\right\}$$

If, therefore, put the observed displacement on the earth's surface be expressed by

$$\left.\begin{aligned} u &= {}_ua_0 + \Sigma_u a_{om}\cos my' + \Sigma_u a_{no}\cos nx' + \Sigma_u b_{om}\sin my' + \Sigma_u b_{no}\sin nx' \\ &\quad + \Sigma_u a_{nm}\cos nx'\cos my' + \Sigma_u a'_{nm}\cos nx'\sin my' \\ &\quad + \Sigma_u b_{nm}\sin nx'\cos my' + \Sigma_u b'_{nm}\sin nx'\sin my', \\ v &= {}_va_0 + \Sigma_v a_{om}\cos my' + \Sigma_v a_{no}\cos nx' + \Sigma_v b_{om}\sin my' + \Sigma_v b_{no}\sin nx' \\ &\quad + \Sigma_v a_{nm}\cos nx'\cos my' + \Sigma_v a'_{nm}\cos nx'\sin my' \\ &\quad + \Sigma_v b_{nm}\sin nx'\cos my', \\ w &= a_0 + \Sigma\, a_{om}\cos my' + \Sigma\, a_{no}\cos nx' + \Sigma\, b_{om}\sin my' + \Sigma\, b_{no}\sin nx' \\ &\quad + \Sigma\, a_{nm}\cos nx'\cos my' + \Sigma\, a'_{nm}\cos nx'\sin my' \\ &\quad + \Sigma\, b_{nm}\sin nx'\cos my' + \Sigma\, b'_{nm}\sin nx'\sin my', \end{aligned}\right\} \quad \ldots\ldots (8)$$

we get the following relations between the coefficients in the series of the displacement and the pressure :

$$\left.\begin{aligned} &{}_ua_o = {}_va_o = {}_ua_{om} = {}_va_{no} = {}_vb_{no} = 0, \quad {}_ua_{no} = b_{no}, \quad {}_ub_{no} = -a_{uo}, \\ &{}_va_{om} = b_{om}, \quad {}_vb_{om} = -a_{om}, \quad A_o = \frac{a_o}{HI_0}, \quad B_{om} = \frac{b_{om}}{HI_2}\,; \\ &A_{om} = \frac{a_{om}}{HI_2} = \frac{{}_vb_{om}}{I_2^{(m)}}, \quad A_{no} = \frac{a_{no}}{HI_1} = \frac{{}_ub_{no}}{I_1^{(n)}}, \quad B_{no} = \frac{b_{no}}{HI_1} = \frac{{}_ua_{no}}{I_1^{(n)}}, \\ &A_{nm} = \frac{a_{nm}}{HI_3} = \frac{{}_ub_{nm}}{I_3^{(n)}} = \frac{{}_va'_{nm}}{I_3^{(m)}}, \quad A'_{nm} = \frac{a'_{nm}}{HI_3} = \frac{{}_ub'_{nm}}{I_3^{(n)}} = \frac{{}_va_{nm}}{I_3^{(m)}}, \\ &B_{nm} = \frac{b_{nm}}{HI_3} = -\frac{{}_ua_{nm}}{I_3^{(m)}} = \frac{{}_vb'_{nm}}{I_3^{(m)}}, \quad B'_{nm} = \frac{b'_{nm}}{HI_3} = \frac{{}_ua_{nm}}{I_3^{(n)}} = -\frac{{}_vb_{nm}}{I_3^{(n)}}. \end{aligned}\right\} \quad \ldots\ldots (9)$$

Though mathematically the equation (6) is divergent, it is of no matter in physical sence, as minute changes in displacement on the earth's surface cannot be attributed to underground pressure distribution in deep layers. First few terms in the series only have physical meaning.

It is also possible to estimate the depth at which the pressure is distributed in this case, as equation (9) gives some relation between coefficients in series of $u$, $v$ and $w$ and in the relation the depth is included. And it is also possible to calculate the three dimentional distribution of pressure.